

Exponent and logarithm rules, taken from

Exponents

$$x^m \cdot x^n = x^{m+n}$$

$$\frac{x^m}{x^n} = x^{m-n}$$

$$(x^m)^n = x^{m \cdot n}$$

$$(xy)^n = x^n y^n$$

$$\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$$

$$x^0 = 1 \quad (x \neq 0)$$

$$x^{-n} = \frac{1}{x^n}$$

$$\frac{x^{-n}}{y^{-m}} = \frac{y^m}{x^n}$$

$$\sqrt[n]{a^m} = a^{m/n}$$

$$\text{Ex: } x^{-\frac{5}{3}} = \frac{1}{\sqrt[3]{x^5}}$$

Logarithms

$$y = \log_b x \quad \text{if and only if} \quad x = b^y$$

$$\log_b 1 = 0$$

$$\log_b b = 1$$

$$\log_b b^x = x$$

$$b^{\log_b x} = x, \quad x > 0$$

$$\text{In particular: } \ln 1 = 0$$

$$\ln e = 1$$

$$\ln e^x = x$$

$$e^{\ln x} = x$$

$$\log_b (xy) = \log_b x + \log_b y$$

$$\log_b \left(\frac{x}{y}\right) = \log_b x - \log_b y$$

$$\log_b x^n = n \log_b x$$

$$\begin{aligned} \text{Ex: } \ln \left(\frac{x^2}{\sin x}\right) &= \ln x^2 - \ln(\sin x) \\ &= 2 \ln x - \ln(\sin x) \end{aligned}$$

$$\log(xy) \neq \log x \cdot \log y$$

caution

$$\log\left(\frac{x}{y}\right) \neq \frac{\log x}{\log y}$$