1. Consider the function $f(x) = (x^2 - 3x + 4)^4$. This is a composite of $h(u) = u^4$ and $u(x) = x^2 - 3x + 4$. Since $h'(u) = 4u^3$, the chain rule gives

$$f'(x) = h'(u(x))u'(x) = 4(x^2 - 3x + 4)^3(2x - 3).$$

2. The function $f(x) = \ln(22 + \sin(7x))$ is the composite of two functions $g(u) = \ln(u)$ and $u(x) = 22 + \sin(7x)$.

$$f'(x) = g'(u(x))u'(x) = \frac{1}{(22 + \sin(7x))} \cdot (0 + 7\cos(7x)) = \frac{7\cos(7x)}{22 + \sin(7x)}.$$

3. Consider the function $f(x) = e^{-6x}(3x + 15)^2 + \ln(x^5 - 10)$. Use the product rule for the first term, x, and then chain rule.

$$\begin{aligned} f'(x) &= e^{-6x}(2)(3x + 15)^1(3) - 6e^{-6x}(3x + 15)^2 + \frac{1}{(x^5 - 10)}5x^4 \\ &= -6e^{-6x}(3x + 15)(3x + 14) + \frac{5x^4}{x^5 - 10}. \end{aligned}$$

4. Consider the function $f(x) = x\sin(x^2 - 7)$. Differentiation uses the product with the second factor requiring the chain rule for its derivative:

$$f'(x) = x\cos(x^2 - 7)(2x) + 1 \cdot \sin(x^2 - 7) = 2x^2\cos(x^2 - 7) + \sin(x^2 - 7).$$

5. Consider $f(x) = \left(x^2 - e^{-x^3}\right)^{-1} + \frac{1}{x^2+4} = \left(x^2 - e^{-x^3}\right)^{-1} + (x^2 + 4)^{-1}$. Application of the chain rule gives:

$$\begin{aligned} f'(x) &= -1\left(x^2 - e^{-x^3}\right)^{-2}(2x - e^{-x^3}(-3x^2)) - 1(x^2 + 4)^{-2}(2x) \\ &= \frac{-(2x + 3x^2e^{-x^3})}{(x^2 - e^{-x^3})^2} - \frac{2x}{(x^2 + 4)^2}. \end{aligned}$$

6. Consider the function $f(x) = (x^4 - \cos(4x^5))^6$. From the chain rule the derivative is

$$\begin{aligned} f'(x) &= 6\left(x^4 - \cos(4x^5)\right)^5(4x^3 + \sin(4x^5)(4 \cdot 5x^4)) \\ &= 24x^3(1 + 5x\sin(4x^5))\left(x^4 - \cos(4x^5)\right)^5. \end{aligned}$$

7. Consider the function $f(x) = \frac{1}{\sin^2(x^3)} = (\sin(x^3))^{-2}$. Rewrite the function,

$$f'(x) = -2(\sin(x^3))^{-3}\cos(x^3) \cdot 3x^2 = -\frac{6x^2\cos x^3}{\sin^3(x^3)}.$$

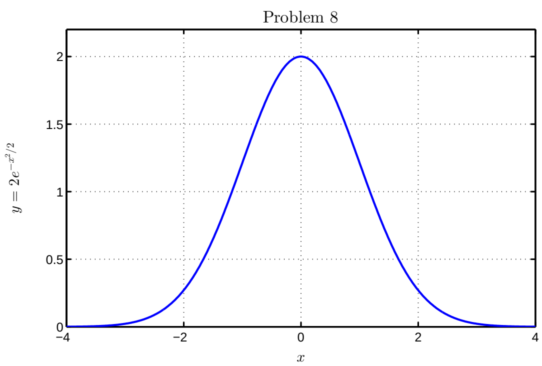
8. Consider the function $y = 2e^{-\frac{x^2}{2}}$. By the chain rule the derivative is

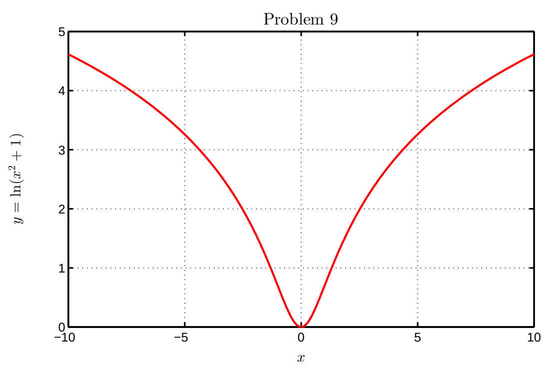
$$y' = 2e^{-\frac{x^2}{2}}\left(\frac{-2x}{2}\right) = -2xe^{-\frac{x^2}{2}}.$$

The product rule followed by the chain rule gives the second derivative as

$$y'' = -2x \cdot e^{-\frac{x^2}{2}}(-x) - 2e^{-\frac{x^2}{2}} = 2(x^2-1)e^{-\frac{x^2}{2}}.$$

The y-intercept satisfies $x = 0$ or $y(0) = 2e^0 = 2$. The x-intercept would satisfy $y = 0 = 2e^{-x^2/2}$, which does not exist, since the exponential function is always positive. Since the exponential function decays very rapidly as $x \to \pm\infty$, there is a horizontal asymptote with $y = 0$. The critical points are where $y' = 0$, or when $x_c = 0$. This is the y-intercept $(0, 2)$. The points of inflection occur where $y'' = 0$, so $(x^2 - 1) = 0$. Thus, $x_{1p} = -1$ with $y(x_{1p}) = 2e^{-\frac{1}{2}} \approx 1.21306$. Similarly, $x_{2p} = 1$ with $y(x_{2p}) = 2e^{-\frac{1}{2}} \approx 1.21306$. This is an even function, reflected across the y-axis. The graph is shown below to the left.





9. The function $y = \ln(x^2+1)$, is the composite of two functions $g(u) = \ln(u)$ and $u(x) = x^2+1$. The derivative satisfies

$$y' = \frac{1}{x^2+1}(2x) = \frac{2x}{x^2+1} = 2x(x^2+1)^{-1}.$$

The second derivative uses the quotient rule giving

$$y'' = \frac{2(x^2+1) - 2x(2x)}{(x^2+1)^2} = \frac{2(1-x^2)}{(x^2+1)^2}.$$

The y-intercept occurs when $y(0) = \ln(0 + 1) = 0$, giving the only intercept (x and y) as $(0, 0)$. There are no asymptotes, since the domain is all x (no vertical) and the logarithmic function is unbounded (no horizontal). Critical points occur when $y' = 0$, so the numerator $2x = 0$ or $x_c = 0$. Thus, there is a minimum at $(0, 0)$. There are points of inflection where $y'' = 0 = (1 - x^2)$ so $x_{1p} = -1$ and $y(x_{1p}) = \ln(2) \approx 0.69315$, with $x_{2p} = 1$ and $y(x_{2p}) = \ln(2)$. This is an even function. The graph is above to the right.

10. Consider the function $y = \frac{10x}{(1+0.1x)^2}$. We apply the quotient and chain rule to yield:

$$y' = 10\frac{(1+0.1x)^2 \cdot 1 - 2(1+0.1x)^1(0.1)x}{(1+0.1x)^4} = \frac{10-x}{(1+0.1x)^3} = (10-x)(1+0.1x)^{-3}.$$

The product rule and chain rule can be used for the second derivative:

$$\begin{aligned} y'' &= (10-x)(-3)(1+0.1x)^{-4}(0.1) - 1(1+0.1x)^{-3} \\ &= -(3(1-0.1x)+(1+0.1x))(1+0.1x)^{-4} \;=\; \frac{0.2(x-20)}{(1+0.1x)^4} \end{aligned}$$

The y-intercept satisfies $y(0)=0$, so the origin is both the x and y-intercept. The domain is $x \neq -10$, which gives a vertical asymptote at $x=-10$. Since the leading power of the denominator exceeds the leading power of the numerator, there is a horizontal asymptote at $y=0$. At the critical points $y'=0$, so $x_c = 10$ and $y(x_c) = \frac{10\cdot 10}{(1+0.1\cdot 10)^2} = 25$, which is a maximum. The point of inflection is where $y''=0$, so $x_i = 20$ and $y(x_i) = \frac{10\cdot 20}{(1+0.1\cdot 20)^2} = \frac{200}{9} \approx 22.22$. This function is neither even nor odd. The graph is below.