1. With the functions $f(t) = t^2 + 6t$ and $g(t) = t - 5$, we have:

a. When $t = 0$, then $f(0) = (0)^2 + 6(0) = 0$. For $t = 2$, $f(2) = (2)^2 + 6(2) = 16$.
When $t = -2$, then $g(-2) = (-2) - 5 = -7$. For $t = 3$, $g(3) = (3) - 5 = -2$.

b. Consider $f(g(t)) = (t-5)^2 + 6(t-5) = t^2 - 4t - 5$.
Similarly, $g(f(t)) = t^2 + 6t - 5$.

c. Evaluating the composite functions, $f(g(1)) = (1)^2 - 4(1) - 5 = -8$ and $g(f(1)) = (1)^2 + 6(1) - 5 = 2$.

2. With $f(x) = -x^2 + 3x + 5$, we have $f(0) = -(0)^2 + 3(0) + 5 = 5$,

$f(-\frac{1}{4}) = -(-\frac{1}{4})^2 + 3(-\frac{1}{4}) + 5 = \frac{67}{16}$,

$f(a) = -(a)^2 + 3(a) + 5 = -a^2 + 3a + 5$,

$f(\frac{1}{d}) = -(\frac{1}{d})^2 + 3(\frac{1}{d}) + 5 = -\frac{1}{d^2} + \frac{3}{d} + 5$.

3. With $f(x) = \frac{5x+4}{x-4}$, we have $f(0) = \frac{0+4}{0-4} = -1$,

$$f(-\tfrac{1}{3}) = \frac{5(-\frac{1}{3}) + 4}{-\frac{1}{3} - 4} = -\frac{7}{13},$$

$f(a) = \frac{5a+4}{a-4}$,

$$f(\tfrac{2}{d}) = \frac{5(\frac{2}{d}) + 4}{\frac{2}{d} - 4} = \frac{5 + 2d}{1 - 2d}.$$

4. With $f(x) = 1 - 5x^2$, we have $f(x+h) = 1 - 5(x+h)^2$,

$f(x+h) - f(x) = 1 - 5(x+h)^2 - 1 + 5x^2 = -5h(2x+h)$,

$\frac{f(x+h)-f(x)}{h} = -5(2x+h)$.

5. With $f(x) = \dfrac{5}{x+5}$, we have $f(x+h) = \dfrac{5}{(x+h+5)}$,

$$f(x+h) - f(x) = \frac{5}{(x+h+5)} - \frac{5}{x+5} = \frac{-5h}{(x+h+5)(x+5)},$$

$$\frac{f(x+h) - f(x)}{h} = \frac{-5}{(x+h+5)(x+5)}.$$

6. With $f(x) = \dfrac{4}{x^2}$, we have $f(x+h) = \dfrac{4}{(x+h)^2}$,

$$f(x+h) - f(x) = \frac{4}{(x+h)^2} - \frac{4}{x^2} = \frac{-4h(h+2x)}{x^2(x+h)^2},$$

$$\frac{f(x+h)-f(x)}{h} = \frac{-4(h+2x)}{x^2(x+h)^2}.$$

7. a. With $f(x) = 3x^2 + 3x + 4$ and $g(x) = 2x - 6$, the compositions are

$$f(g(x)) = 3(2x-6)^2 + 3(2x-6) + 4 = 12x^2 - 66x + 94,$$

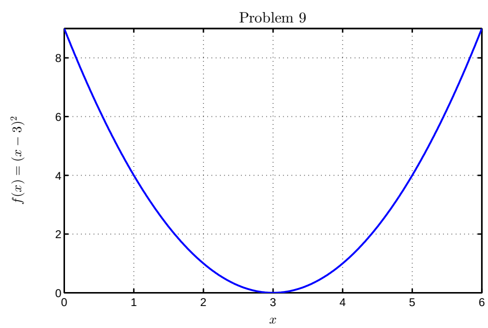
$$g(f(x)) = 2(3x^2 + 3x + 4) - 6 = 6x^2 + 6x + 2\ .$$

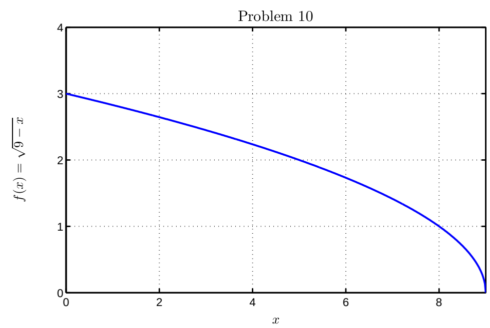
b. Evaluating the compositions at $x = 2$, we have $f(g(2)) = 12(2)^2 - 66(2) + 94 = 10$ and $g(f(2)) = 6(2)^2 + 6(2) + 2 = 38$.

8. a. The function $f(x) = 9 - x^2$ is a parabola pointing down with its vertex at $(0,9)$. Since the maximum is at the vertex, the range of this function is $(-\infty, 9]$.

b. With the range restricted to $f(x) > 0$, we solve $9 - x^2 > 0$. This yields $x^2 < 9$ or $-3 < x < 3$, so $x \in (-3, 3)$.

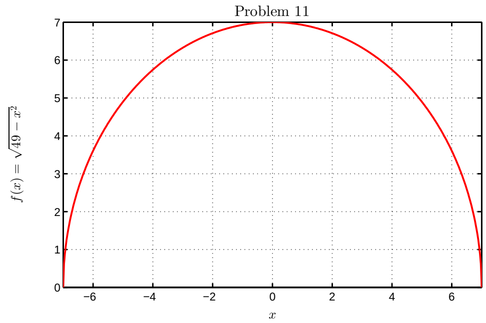
9. For the function, $f(x) = (x-3)^2$, the **domain** is all x, so $x \in (-\infty, \infty)$. The function is a parabola pointing up with a vertex at $(3,0)$, so the **range** is $[0, \infty)$. This function has no symmetry about the origin or y-axis, so is neither even nor odd. The graph is shown below to the left.

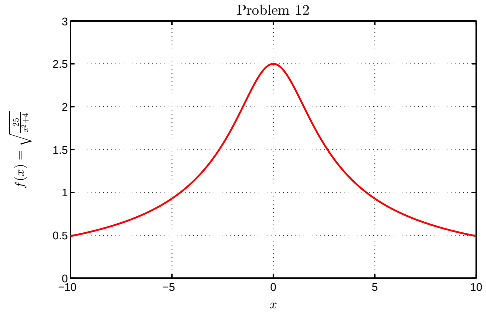




10. For $y = \sqrt{9-x}$, the domain is determined by finding where the quantity under the radical is non-negative or $9 - x \geq 0$, so the **domain** is $x \in (-\infty, 9]$. This function is defined for positive values of y, so the **range** is $[0, \infty)$. The function has neither even nor odd symmetry. The graph is above to the right

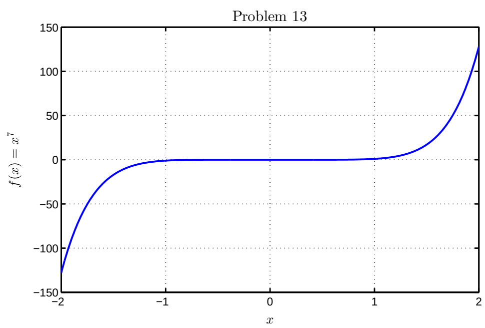
11. For $y = \sqrt{49 - x^2}$, the domain is determined by finding where the quantity under the radical is non-negative or $49 - x^2 \geq 0$, so the **domain** is $x \in [-7, 7]$. This function creates a semi-circle. The maximum inside the domain is $y = 7$, which when combined with the function being non-negative, gives the **range** is $[0, 7]$. The graph is symmetric around the y-axis so shows **even** symmetry. The graph is below to the left.

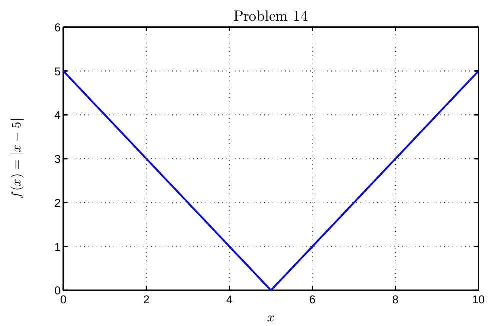




12. For $f(x) = \sqrt{\dfrac{25}{x^2+4}}$, the **domain** is all x, so $x \in (-\infty, \infty)$. The function approaches 0 for large or small x, and the maximum value is when $x = 0$ with $f(0) = \sqrt{\frac{25}{4}} = \frac{5}{2}$. It follows that the **range** is $(0, \frac{5}{2}]$. The graph is symmetric about the y-axis, so the function is **even**. The graph is above to the right.

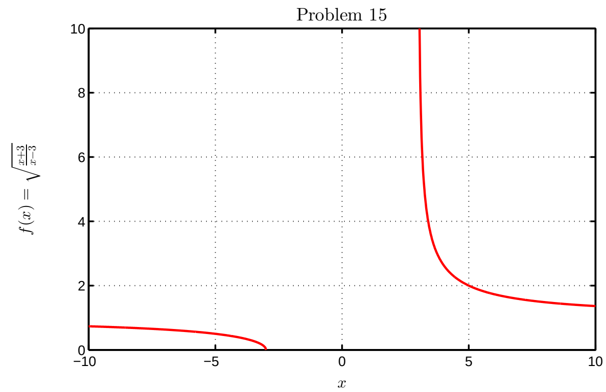
13. For $f(x) = x^7$, the **domain** is all x, so $x \in (-\infty, \infty)$. The **range** is also $(-\infty, \infty)$. The graph is symmetric about the origin, so the function is **odd**. The graph is below to the left.





14. For $f(x) = |x - 5|$, the **domain** is all x, so $x \in (-\infty, \infty)$. The absolute value restricts the range to non-negative values, so the **range** is $[0, \infty)$. This function has no symmetry about the origin or y-axis. The graph is above to the right.

15. For $f(x) = \sqrt{\dfrac{x+3}{x-3}}$, the domain is determined by finding where the denominator is non-zero, so $x \neq 3$, and the quantity under the radical is non-negative. We solve $\frac{x+3}{x-3} \geq 0$, then either $x > 3$ or $x \leq -3$. It follows that the **domain** is $x \in (-\infty, -3] \cup (3, \infty)$. From the domain, we find the **range** is $[0, 1) \cup (1, \infty)$. This function has no symmetry. The graph is shown below.


Problem 15
$f(x) = \sqrt{\frac{x+3}{x-3}}$
x
10
8
6
4
2
0
−10
−5
0
5
10