1. Consider the function $f(x) = \frac{x^3-\ln(x)}{1-x^2} + \frac{2}{x^2} = \frac{x^3-\ln(x)}{1-x^2} + 2x^{-2}$. Applying the quotient rule to the first part and the power rule to the second, we have:

$$\begin{aligned} f'(x) &= \frac{(1-x^2)(3x^2-\frac{1}{x}) - (x^3-\ln(x))(-2x)}{(1-x^2)^2} - 2\cdot 2x^{-3} \\ &= \frac{(1-x^2)(3x^2-\frac{1}{x}) + 2x(x^3-\ln(x))}{(1-x^2)^2} - 4x^{-3}. \end{aligned}$$

2. Consider the function $f(x) = \frac{x^2-e^{-x}}{3x+1} + xe^{-x}$. Applying the product and quotient rule to this function, we have:

$$\begin{aligned} f'(x) &= \frac{(3x+1)(2x+e^{-x}) - (x^2-e^{-x})(3)}{(3x+1)^2} + (-xe^{-x} + 1\cdot e^{-x}) \\ &= \frac{(3x+1)(2x+e^{-x}) - 3(x^2-e^{-x})}{(3x+1)^2} + (1-x)e^{-x}. \end{aligned}$$

3. Consider the function $f(x) = \frac{\sqrt{x}}{2+x} - \frac{1}{e^{3x}} = \frac{x^{\frac{1}{2}}}{2+x} - e^{-3x}$. From our rules of differentiation, we have

$$\begin{aligned} f'(x) &= \frac{(2+x)\left(\frac{1}{2}x^{-\frac{1}{2}}\right) - (1)(x^{\frac{1}{2}})}{(2+x)^2} - (-3)e^{-3x} \\ &= \frac{2+x-2x}{2(2+x)^2\sqrt{x}} + 3e^{-3x} = \frac{2-x}{2(2+x)^2\sqrt{x}} + 3e^{-3x} \end{aligned}$$

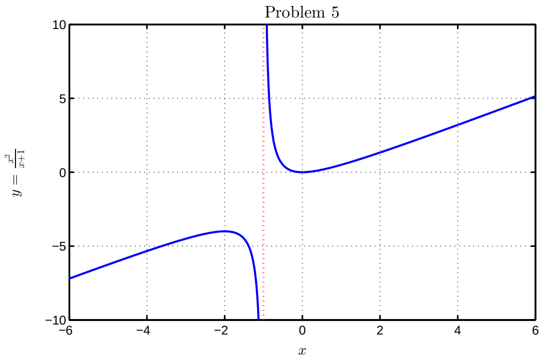
4. Consider the function $f(x) = \frac{8e^{-2x}}{12+\cos(2x)}$. The quotient rule gives the derivative

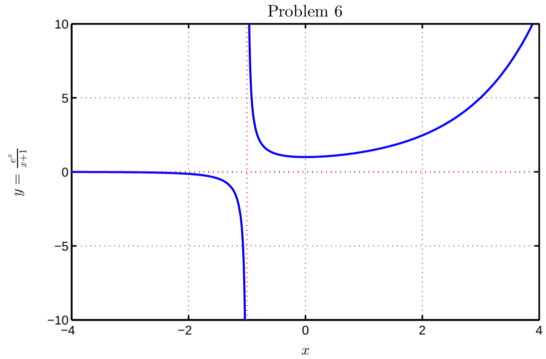
$$\begin{aligned} f'(x) &= \frac{(12+\cos(2x))(8(-2)e^{-2x}) - (8e^{-2x})(-2\sin(2x))}{(12+\cos(2x))^2} \\ &= \frac{16(\sin(2x)-\cos(2x)-12)e^{-2x}}{(12+\cos(2x))^2}. \end{aligned}$$

5. Consider the function $y(x) = \frac{x^2}{x+1}$. The quotient rule finds the derivative

$$y'(x) = \frac{(x+1)(2x) - (1)x^2}{(x+1)^2} = \frac{x(x+2)}{(x+1)^2}$$

The x-intercept occurs where $y = 0$, which is $x = 0$, so the x and y-intercept is the origin, $(0,0)$. There is no horizontal asymptote, because the exponent in the numerator is higher than that in the denominator. The vertical asymptote occurs when the denominator is zero, or $x+1 = 0$ so $x = -1$. At the critical points, $y' = 0 = \frac{x(x+2)}{(x+1)^2}$, so $x(x+2) = 0$. Thus, $x_{1c} = -2$ and $x_{2c} = 0$. The y values are $y_{1c} = \frac{(-2)^2}{-2+1} = -4$ and $y_{2c} = \frac{0^2}{0+1} = 0$. It follows that $(-2,-4)$ is a maximum and $(0,0)$ is a minimum. The graph appears below to the left.





6. Consider the function $y(x) = \frac{e^x}{x+1}$. The quotient rule finds the derivative

$$y' = \frac{(x+1)e^x - (1)e^x}{(x+1)^2} = \frac{xe^x}{(x+1)^2}.$$

The x-intercept occurs when $y = 0$. Since the exponential function is not zero, there are no x-intercepts. The y-intercept occurs when $x = 0$ or $y(0) = \frac{e^0}{0+1} = 1$, so the y-intercept is $(0, 1)$. Since the denominator $x + 1 = 0$ when $x = -1$, this is a vertical asymptote. From the limit below,

$$\lim_{x \to -\infty} \frac{e^x}{x+1} = 0,$$

so there is a horizontal asymptote to the left at $y = 0$. The critical point satisfies $y' = 0$, so $0 = xe^x$ or $x_c = 0$. Since $y(x_c) = 1$, we have $(0, 1)$ is a minimum. The graph appears above on the right.

7. Consider the function $y(x) = \frac{x^2-2x+2}{x-1}$. The quotient rule finds the derivative

$$y' = \frac{(x-1)(2x-2) - (1)(x^2 - 2x + 2)}{(x-1)^2} = \frac{x(x-2)}{(x-1)^2}.$$

The x-intercept occurs when $y = 0$, but $x^2 - 2x + 2 = 0$ has no real solution, so there is no x-intercept. The y-intercept occurs when $x = 0$, so $y(0) = -2$. There is no horizontal asymptote, as the highest exponent in the numerator is larger than that in the denominator. There is a vertical asymptote where the denominator is zero, or $x = 1$. The critical points satisfy $y' = 0$, so $x(x - 2) = 0$. It follows that $x_{1c} = 0$ and $y(x_{1c}) = -2$, which gives a maximum at $(0, -2)$. Similarly, $x_{2c} = 2$ and $y(x_{2c}) = 2$, which gives a minimum at $(2, 2)$. The graph is shown below on the left.