

Problems- Compound interest

Compound Interest:

$$P = P_0 \left(1 + \frac{r}{n} \right)^{nt}$$

P_0 : the principal, amount invested
 r : the rate, (in decimal form)

P : the new balance
 n : the number of times it is compounded.

t : the time in years

Ex1: Suppose that \$5000 is deposited in a saving account at the rate of 6% per year. Find the total amount on deposit at the end of 4 years if the interest is:

$$P_0 = \$5000, r = 6\%, t = 4 \text{ years}$$

a) compounded annually, $n = 1$: $P = 5000(1 + 0.06/1)^{(1)(4)} = 5000(1.06)^{(4)} = \6312.38

b) compounded semiannually, $n = 2$: $P = 5000(1 + 0.06/2)^{(2)(4)} = 5000(1.03)^{(8)} = \6333.85

c) compounded quarterly, $n = 4$: $P = 5000(1 + 0.06/4)^{(4)(4)} = 5000(1.015)^{(16)} = \6344.93

d) compounded monthly, $n = 12$: $P = 5000(1 + 0.06/12)^{(12)(4)} = 5000(1.005)^{(48)} = \6352.44

e) compounded daily, $n = 365$: $P = 5000(1 + 0.06/365)^{(365)(4)} = 5000(1.00016)^{(1460)} = \6356.12

For Compounded Annually where $n = 1$: $P = P_0 \left(1 + \frac{r}{n} \right)^{nt} = P_0 (1 + r)^t$

Continuous Compound Interest:

Continuous compounding means compound every instant, consider investment of 1\$ for 1 year at 100% interest rate. If the interest rate is compounded n times per year, the compounded amount as we saw before is given by:

$$P = P_0 (1 + r/n)^{nt}$$

The following table shows the compound interest that results as the number of compounding periods increases:

$$P_0 = \$1; r = 100\% = 1; t = 1 \text{ year}$$

Compounded	n	Compound amount
annually	1	$(1+1/1)^1 = \$2$
monthly	12	$(1+1/12)^{12} = \$2.6130$
daily	360	$(1+1/360)^{360} = \$2.7145$
hourly	8640	$(1+1/8640)^{8640} = \$2.71812$
each minute	518,400	$(1+1/518,400)^{518,400} = \2.71827

As the table shows, as n increases in size, the limiting value of P is the special number $e = 2.71828$

If the interest is compounded continuously for t years at a rate of r per year, then the compounded amount is given by:

$$P = P_0 \cdot e^{rt}$$

Ex2: Suppose that \$5000 is deposited in a saving account at the rate of 6% per year. Find the total amount on deposit at the end of 4 years if the interest is compounded continuously. (compare the result with example 1)

$$P_0 = \$5000, r = 6\%, t = 4 \text{ years}$$

$$P = 5000 \cdot e^{(0.06)(4)} = 5000 \cdot (1.27125) = \$6356.24$$

Ex3: Find the amount to be invested at a rate of 8% compounded continuously in order to get \$12,930 in 6 years.

$$P = \$12,930, \quad r = 0.08, \quad t = 6 \text{ years.}$$

$$\$12,930 = P_0 \cdot e^{(0.08)(6)}, \text{ then } P_0 = \$8000$$

The Growth, Decline:

<p>Continuous Growth: $\rightarrow P = P_0 \cdot e^{rt}$</p> <p>Annual Growth: $\rightarrow P = P_0(1+r)^t = P_0 a^t$ where $a > 1$</p>	<p>Continuous Decline: $\rightarrow P = P_0 \cdot e^{-rt}$</p> <p>Annual Decline: $\rightarrow P = P_0(1-r)^t = P_0 b^t$ where $0 < b < 1$</p>
<p>Doubling Time $= \frac{\ln 2}{r}$ (for continuous growth) $= \frac{\ln 2}{\ln(1+r)}$ (for annual growth)</p> <p>Half-life $= \frac{\ln 2}{r}$</p>	

Ex4: The growth rate in a certain country is 15% per year. Assuming continuous growth :

- a) If the population is 100,000 now, find the new population in 5 years.
- b) When will the 100,000 double itself?

Ex5: In 1965 the price of a math book was \$16. In 1980 it was \$40. Assuming the continuous growth :

- a) Find r and write the equation.
- b) Find the cost of the book in 1985.
- c) After when will the cost of the book be \$32 ?

Ex6: A couple want an initial balance to grow to \$ 211,700 in 5 years. The interest rate is compounded continuously at 15%. What should be the initial balance?

Ex7: The population of a city was 250,000 in 1970 and 200,000 in 1980 (*Decline*). Assuming the population is decreasing continuously, find the population in 1990.

Ex8: At what rate compounded annually will an investment of \$2100 accumulate to \$3400 by the end of 6 years

Ex9: How long does it take amount to double at 8.5% compounded: annually, continuously?

Ex10. The population of a certain town is declining exponentially due to immigration. If the population was reduced by 20% after 10 years, find the decline rate.

Ex11. Write the equation of problem 10 but: If only 85% the population are present after 10 years.

Ex12.. The half-life of a certain radioactive substance is 12 days. If there are 10 grams initially:

- a) find the rate.
- b) when will the substance be reduced to 2 grams?

Ex13. Convert the function $P = 400e^{0.05t}$ to the form $P = P_0 a^t$

Ex14. Convert the function $P = 2000(1.08)^t$ to the form $P = P_0 e^t$

- Answers:*
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|---------------------|-------------------------|----------------------------|-----------------------|
| 4. (211700, 4.62) | 5. (6.1%, 54.28, 11.36) | 6. (100,000) | 7. (160,000) |
| 8. (8.36%) | 9. (8.5, 8.15) | 10. (2.23%) | 11. $0.85 = e^{-10t}$ |
| 12. (5.78% , 27.86) | 13. $P = 400(1.0513)^t$ | 14. $P = 2000e^{0.07696t}$ | |