

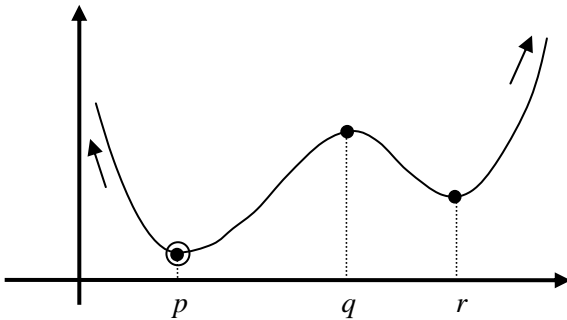


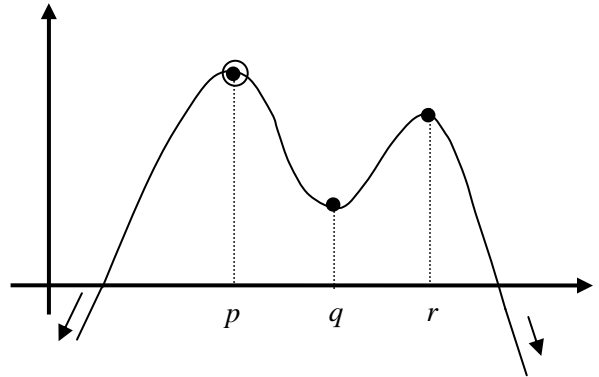
## Supplement

- Local (Relative) Max and Local Min:** where  
 $f'(x) = 0$  and  $f''(x) < 0$  for local max  (slope of tangent line = 0, concave down)  
 $f'(x) = 0$  and  $f''(x) > 0$  for local min  (slope of tangent line = 0, concave up)  
 $f'(x)$  does not exist but  $f(x)$  does
- Global Max and Global Min:** The absolute highest and lowest points of the function including the end points.

### a) Open Interval, No End Points (entire real line):

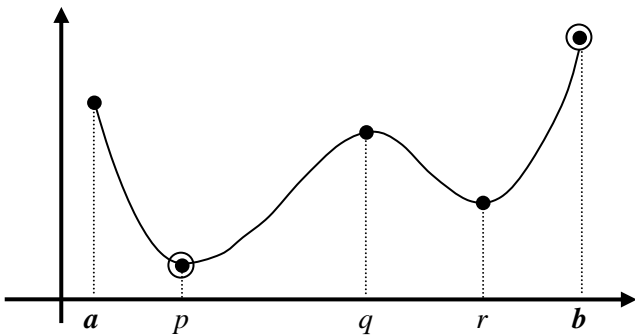


- Local Maximum at:  $q$
- Local Minimum at:  $p$  and  $r$
- No Global (Absolute) Maximum
- Global (Absolute) Minimum at:  $p$

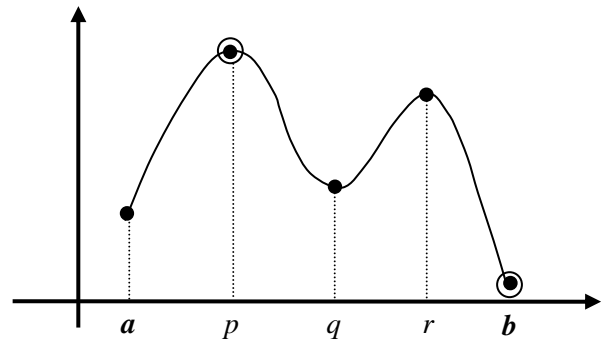


- Local Maximum at:  $p$  and  $r$
- Local Minimum at:  $q$
- Global (Absolute) Maximum at:  $p$
- No Global (Absolute) Minimum

### b) Closed Interval, With End Points such as $a \leq x \leq b$ :



- Local Maximum at:  $a$ ,  $q$  and  $b$
- Local Minimum at:  $p$  and  $r$
- Global (Absolute) Maximum at:  $b$
- Global (Absolute) Minimum at:  $p$



- Local Maximum at:  $p$  and  $r$
- Local Minimum at:  $a$ ,  $b$ , and  $q$
- Global (Absolute) Maximum at:  $p$
- Global (Absolute) Minimum at:  $b$

The following example is similar to problems 18, 19 and 20 on page 186.

**Example 1:** For the function  $f(x) = -x^3 + 3x^2 - 4$  ;  $(-1.5 \leq x \leq 3)$

- a) Find the  $f'$  and  $f''$ .
  - b) Find the critical points.
  - c) Find the inflection points.
  - d) Evaluate  $f$  at its critical points and the endpoints of the given interval. Identify local and global maxima and minima in the interval.
  - e) Graph  $f$
- 

The following examples are similar to the final exam style (keep your work to review for final exam).

**Example 2:** For the function  $f(x) = x^3 - 3x^2 + 6$  ;  $(-1.1 \leq x \leq 2.5)$

- a) Find the  $f'$  and  $f''$ .
- b) Find the critical points.
- c) Find the inflection points.
- d) Use 1<sup>st</sup> or 2<sup>nd</sup> derivative test to classify the critical points as local max or local min.
- e) Find any global max or global min
- f) Sketch a graph of the function.

**Example 3:** For the function  $f(x) = 2x^3 - 6x + 2$  ;  $(-1.5 \leq x \leq 2)$

- a) Find the  $f'$  and  $f''$ .
- b) Find the critical points.
- c) Find the inflection points.
- d) Use 1<sup>st</sup> or 2<sup>nd</sup> derivative test to classify the critical points as local max or local min.
- e) Find any global max or global min
- f) Sketch a graph of the function.

*Answers for Example 3:*

Critical points at  $x = -1$  and  $x = 1$ . Inflection point at  $(0, 2)$

Local Max at  $(-1.5, 4.25)$  ; Global Min at  $(1, -2)$  ; Global Max at  $(-1, 6)$  and  $(2, 6)$

**Example 1 Solution:**

$$f(x) = -x^3 + 3x^2 - 4; \quad (-1.5 \leq x \leq 3)$$

a)  $f'(x) = -3x^2 - 6x$  ;  $f''(x) = -6x - 6$

b) Critical points where  $f'(x) = 0$  , then

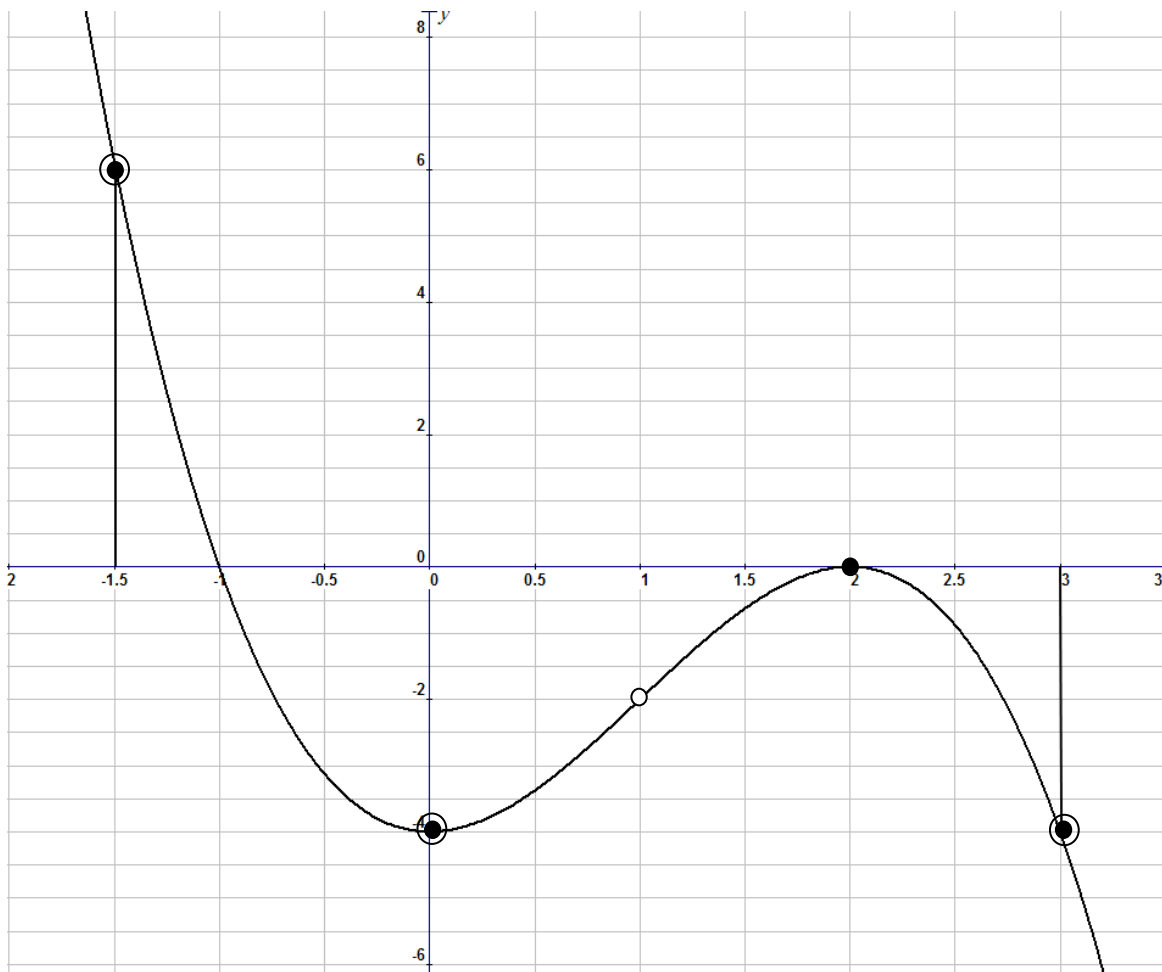
$$-3x^2 - 6x = 0 \quad \text{or} \quad -3x(x - 2) = 0 \rightarrow x = 0 \text{ and } x = 2$$

c) Inflection points where  $f''(x) = 0$  , then

$$-6x - 6 = 0 \quad \text{or} \quad -6(x - 1) = 0 \rightarrow x = 1, y = -2$$

(Substitute  $x = 1$  in the original function of  $f(x) = -x^3 + 3x^2 - 4$  to get  $y = -2$ ).

- |                        |               |                   |                             |
|------------------------|---------------|-------------------|-----------------------------|
| d) $x = 0$             | $\rightarrow$ | $f(0) = -4$       | Global Min at (0,-4)        |
| $x = 2$                | $\rightarrow$ | $f(2) = 0$        | Local Max at (2, 0)         |
| $x = -1.5$ (end point) | $\rightarrow$ | $f(-1.5) = 6.125$ | Global Max at (-1.5, 6.125) |
| $x = 3$ (end point)    | $\rightarrow$ | $f(3) = -4$       | Global Min at (3, -4)       |



**Example 2 Solution:**

$$f(x) = x^3 - 3x^2 + 6; \quad (-1.1 \leq x \leq 2.5)$$

a)  $f'(x) = 3x^2 - 6x$  ;  $f''(x) = 6x - 6$

b) Critical points where  $f'(x) = 0$  , then


$$3x^2 - 6x = 0 \quad \text{or} \quad 3x(x - 2) = 0 \quad \rightarrow \quad x = 0 \quad \text{and} \quad x = 2$$


c) Inflection points where  $f''(x) = 0$  , then

$$6x - 6 = 0 \quad \text{or} \quad 6(x - 1) = 0 \quad \rightarrow \quad x = 1, y = 4$$

(Substitute  $x = 1$  in the original function of  $f(x) = x^3 - 3x^2 + 6$  to get  $y = 4$ ).

d) Local Max and Local Min at the critical points of  $x = 0$  and  $x = 2$ . Substitute each point in the second derivative and check the sign of the second derivative:

$f''(0) = 6(0) - 0 = -6 < 0$ ; concave down  Local Max at  $x = 0$

$f''(2) = 6(2) - 2 = 6 > 0$ ; concave up  Local Min at  $x = 2$

e) Use End Points and critical points to check Global Max and Global Min:

$x = 2 \quad \rightarrow \quad f(2) = 2 \quad \text{Local Min at } (2, 2)$

$x = 0 \quad \rightarrow \quad f(0) = 6 \quad \text{Global Max at } (0, 6)$

$x = -1.1$  (end point)  $\rightarrow \quad f(-1.1) = 1.04 \quad \text{Global Min at } (-1.1, 1.04)$

$x = 2.5$  (end point)  $\rightarrow \quad f(2.5) = 2.875 \quad \text{Local Max at } (2.5, 2.875)$

